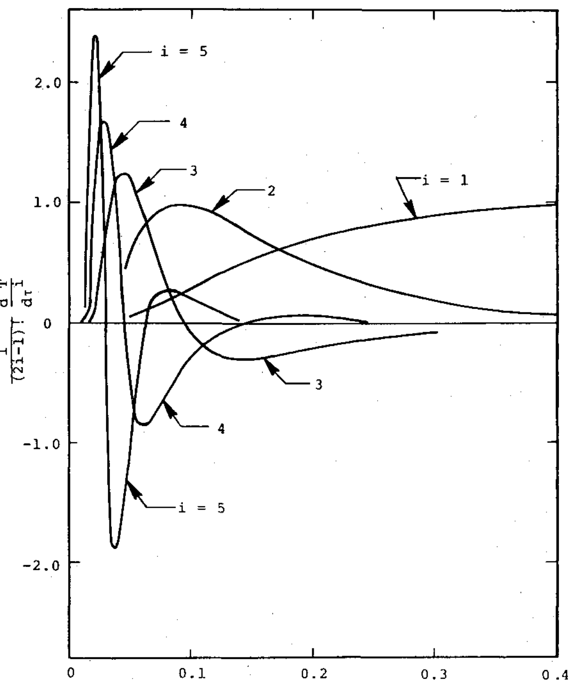

Fig. 1 Derivative terms appearing in Eq. (5) for the insulated surface of a plate heated with a constant flux on one side and insulated on the other.

backward time differences of T_n^* divided by $\Delta\tau$ to the i power.

In order to obtain insight into the relative values of each of the terms in the above summation, the values for the first five i's are plotted in Fig. 1 for a plate heated by a constant heat flux on one side and insulated on the other; the curves are for the insulated surface. Notice that the first derivative term is the only important one for $\tau > 0.4$ but, as the time is decreased, more and *higher* order terms become important. After *any* rapid change in surface heat flux, similar behavior of the derivatives would occur.

As time steps become small ($\Delta\tau = 0.02$ is mentioned[1]), the authors' method implies approximating high-order time derivatives which can be very difficult to do accurately with temperature *measurements* because high-order differences are involved.

The method of Ref. 2 was developed for cases in which the surface flux might change abruptly with time and the maximum information regarding the surface conditions was desired. These conditions require the minimum possible time step. By utilizing future temperature measurements, accurate heat fluxes can be predicted utilizing unsmoothed data (of sufficient accuracy[4]) for small time steps. Additional future measurements may not be needed for extremely accurate data and/or relatively large time steps.

A conclusion of Ref. 1 was that no advantage was gained using the method of Ref. 2 with future temperatures. While this could be true for determining the surface temperature for smoothed data, smoothly varying heat fluxes, and/or large time steps, it is not true in general for determining the surface heat flux.

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Reply by Authors to James V. Beck

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SINCE one of the objectives of our paper[1] was to compare Beck's method with the method we proposed, the comments by Dr. Beck are appreciated. The following remarks are offered in response to his comments.

1) In solving for the net heating rate, the surface temperature is obtained by either method, that of Ref. 1 or Beck's.[2] Thus, if the converged value for the net heating rate is known, so is the surface temperature. The surface heat flux, on the other hand, is known only if the surface properties, such as emmisivity, are known. In dealing with experimental data, the emissivity is usually only approximated and the surface heating rate is given by

$$\dot{q}_{\mathrm{conv}} = \dot{q}_{\mathrm{net}} + \sigma\epsilon(T_s^4 - T_0^4)$$

where $\dot{q}_{\mathrm{conv}}$ is the convective heating rate, q_{net} is the net heating rate, σ is the Stefan-Boltzmann constant, ϵ is the emissivity, T_s is the surface temperature, and T_0 is the surface radiation sink temperature. In any case, it is more valid to discuss surface temperature or net heating than the surface heating rate. Neither is more or less difficult.

The information relating to properties for the linear problem was deleted due to the recommendation of the referees. For the linear model[1] the thermal model consists of 5.08 cm (2 in.) of aluminum with a thermocouple on the backwall. Since this is a linear problem, the surface emittance was set to zero and constant properties were used. The thermal properties were: density = 2851.29 kg/m^3 (178 lb/ft^3); specific heat = 836.80 J/kg-K (0.2 Btu/lb$_m$-°R); and conductivity = 145.28 W/m-K (84 Btu/ft-h-°R). An initial temperature of 294.44 K (530°R) was used, and the backwall was insulated; i.e., adiabatic.

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The heating rate, q, was given by

$$q = \begin{cases} (\text{Btu/ft}^2\text{-s}) & (\text{W/m}^2) & \\ (\tfrac{1}{2}t)5 & 1.1348931 \times 10^4 (\tfrac{1}{2}t)5 & (0 \leq t < 24) \\ (24 - \tfrac{1}{2}t)5 & 1.1348931 \times 10^4 (24 - \tfrac{1}{2}t)5 & (24 < t \leq 48) \\ 0 & 0 & (48 < t) \end{cases}$$

where t is the transient time in seconds. As with the case that Beck reported, it was assumed that the thermocouple was on the backwall of the aluminum. A time step of 2 seconds was used to correspond to the dimensionless time step of 0.05 used by Beck. The results of this analytical investigation were summarized in Table 2.

The results presented in Table 3 (Ref. 1) were also for analytical data using a nonlinear model with both temperature and pressure dependent properties. Since a known heating rate was imposed on the surface, the average error comparison is valid.

2) While the method proposed was using a *single* interior thermocouple, no restriction was placed on receiving the data at evenly spaced time intervals. In addition, the computational time step used is based on the requirements to accurately predict the surface heating conditions, not the data sampling rate. Also, data is not always available at evenly spaced time intervals.

While Beck's method may be more general than the method proposed in Ref. 1, frequently only one thermocouple is available for predicting the surface conditions. This may be due to any number of reasons.

The only problem Ref. 1 attempted to solve was for materials with temperature- and pressure-dependent properties. The linear thermal model was of interest only to verify the correct implementation of Beck's method. For this class of problems the user is advised to use an iterative method, however, if no iterations are required this should be the subject for a paper which demonstrates even greater efficiency.

The efficiency comparison for each case presented was made on the same basis: a counter set in the computer program to count the number of times the tri-diagonal matrix was solved. The results of this counter were the values reported. The algorithm for Beck's method was the one presented in his paper.[2]

3) It is difficult to relate a quantity such as the dimensionless time to real properties. It can only be approximated from the property values that are valid in the temperature and pressure range at a given time.

In a limited sense smoothing does use future and past time information. However, the advantage of using orthogonal polynomials lies in the characteristic that, in the least-squares analysis, it appears as if each parameter were the only one used. The problems mentioned by Beck are correct, however, there are advantages. If the data is too noisy to be of use, the computer run can be terminated and save the computer time that would be wasted on analyzing bad data. If the data is acceptable, computer time can be saved by using the method of Ref. 1.

Whether one attempts to converge to known temperatures or to the net heating rate based on known temperatures, the problems are equivalent. The only difference appears in the magnitude required for convergence. The difference in the magnitude of the convergence criteria for both methods was shown.[1]

The problems mentioned for a rapidly varying heat flux are correct; however, recourse is usually made to use data and time steps appropriate to the problem to be solved. It should be noted that the least-squares fitting of the data can be made as fine as the data allows. The interpolated values obtained from this fit can be made at time intervals convenient for the thermal model, i.e., at the time steps used for the surface heating calculations. This can reduce the total computer time requirements considerably.

In addition, there is less smearing or averaging of the data using the least-squares than for Beck's method of smoothing. This tends to provide more accurate results. With Beck's method the more future times that are used the more the data and the answers will be smeared in the analysis. This difference is not usually considered significant for analysis purposes.

As mentioned in item 1 there is no substantial difference in reporting surface temperatures and heating rates. Both are simultaneously available. Since experimental heat fluxes were not available for comparison, surface temperatures were reported instead of fluxes. At least the temperatures could be compared to the measured thermocouple data. As was mentioned above, it is difficult to discuss dimensionless time for a nonlinear problem.

The main advantage in using the method of Ref. 1 lies in reducing the total computer time required for analysis of the surface conditions for a nonlinear problem. The disadvantage is in building the smoothing algorithm. The main advantage of Beck's method is in being able to analyze the heating conditions without building a smoothing routine — it is inherent in his method. The disadvantage is due to the computational expense.

Since it takes the same amount of time to read the data tape for either method and the computational expense of smoothing the data is small, Ref. 1 provides significant savings in computer time.

References

[1] Williams, S.D. and Curry, D.M., "An Analytical and Experimental Study for Surface Heat Flux Determination", *Journal of Spacecraft and Rockets*, Vol. 14, Oct. 1977, pp. 632-637.

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